Impulse Virtual Distortion Method for Single Damage Identification in Structures

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Abstract

The Impulse Virtual Distortion Method *IVDM* is developed from the Virtual Distortion Method *VDM* (static case) and dedicated to remodelling of structures under dynamic load as well as adapted to damage identification. The latter application of the *IVDM* is based on the analysis of perturbation in vibration response caused by structural defects. In general case, when a few defects are detected, the method is very costly numerically and time-consuming. In order to avoid this problem, the presented approach is concentrated on one damage in the considered structure only. This paper describes the fundamentals of VDM and IVDM. Then the methodology of the single damage identification is presented.

Keywords: Impulse Virtual Distortion Method, damage identification, structural remodelling

1. Introduction

There are some industrial structures (such as pipelines, storage tanks, suspension bridges), the technical condition of which should be monitored, especially zones prone to damage in corrosive environment. The Impulse Virtual Distortion Method is dedicated to remote monitoring of the above-mentioned structural health.

An extensive presentation of the Virtual Distortion Method (strategies of control and design) was given in works [1] and [2]. A comprehensive idea of the application of the Impulse Virtual Distortion Method (IVDM) to the damage detection is given in [3] and [6].

The Impulse Virtual Distortion Method with reference to damage detection was presented in general case (simultaneously a few defects possible) in [5], however is it very costly numerically and time-consuming even for a non-complex structure. Now, in this paper we assume that a structure is initially healthy and we are able to perform its continuous monitoring and observe the first occurred damage. Assuming one defect to be detected, the time of calculation and memory requirements have been significantly reduced.

2. Basics of *VDM* and *IVDM*

2.1. *Virtual Distortion Method*

The following notions play the most substantial role in VDM:

- the **virtual distortion** ε_i^0, modelling structural modification, is an initial strain introduced in a structural element,
- the **unit distortion** is the virtual distortion that would cause a unit strain in an element (out of structure),
- the **compensative load** is the self equilibrated load, applied to the nodes of an element, that is equivalent to unit distortion,
- the **strain influence matrix** D_{ij} and the **general influence matrix** $\breve{D}_{\alpha i}$, which contain respectively the generalized strains and a linear combination of the generalized displacement obtained for unit distortion imposed successively in structural elements.

Let us denote as $\overset{\mathrm{L}}{f}_\alpha$ (or $\overset{\mathrm{L}}{\varepsilon}_i$) the original response of a considered structure — linear combination of any generalized displacements (or generalized strains). Assuming structural modification (in one location) and having calculated influence matrices and the distortions, we can determine the updated response as follows:

$$\varepsilon_i = \overset{\mathrm{L}}{\varepsilon}_i + \overset{\mathrm{R}}{\varepsilon}_i = \overset{\mathrm{L}}{\varepsilon}_i + \sum_j D_{ij}\,\varepsilon_j^0, \tag{1}$$

$$f_\alpha = \overset{\mathrm{L}}{f}_\alpha + \overset{\mathrm{R}}{f}_\alpha = \overset{\mathrm{L}}{f}_\alpha + \sum_i \breve{D}_{\alpha i}\,\varepsilon_i^0. \tag{2}$$

The second terms ($\overset{\mathrm{R}}{\varepsilon}_i$ and $\overset{\mathrm{R}}{f}_\alpha$) in the above equations describe the structural modification. The sums concern elements in which a damage can occur. In general, there are a few components of distortion for an element (for truss element — one component, for beam element — three components, etc.).

2.2. *Modelling of Structural Parameters*

Let us define the vector of stiffness modification $\mu_i = k_i'/k_i$, where k_i and k_i' are stiffness parameters of an intact and modified structure (e.g. Young's modulus). We consider the structure modified in some locations, which we model by the original structure with imposed distortions in those locations. We postulate that the structure modelled by distortions and the modified structure are identical in the sense of generalized strains and stresses. The general stresses and strains in some member of the structure are given by Eqn (3) and (4).

$$S_i = k_i \overset{\mathrm{L}}{\varepsilon}_i + \sum_j (D_{ij} - \delta_{ij})\, \varepsilon_j^0, \tag{3}$$

$$\varepsilon_i = \overset{\mathrm{L}}{\varepsilon}_i + \sum_j D_{ij}\, \varepsilon_j^0. \tag{4}$$

2.3. Impulse Virtual Distortion Method

The Impulse Virtual Distortion Method is used for dynamic problem, so now the virtual distortion and impulse influence matrices are time-dependent. The updated general dynamic response is computed analogously to the Eqn (2):

$$f_\alpha(t) = \overset{\mathrm{L}}{f}_\alpha(t) + \overset{\mathrm{R}}{f}_\alpha(t) = \overset{\mathrm{L}}{f}_\alpha(t) + \sum_i \sum_{\tau=0}^{t} \breve{D}_{\alpha i}(t-\tau)\, \varepsilon_i^0(t), \tag{5}$$

and strains analogously to the Eqn (1):

$$\varepsilon_i(t) = \overset{\mathrm{L}}{\varepsilon}_i(t) + \overset{\mathrm{R}}{\varepsilon}_i(t) = \overset{\mathrm{L}}{\varepsilon}_i(t) + \sum_j \sum_{\tau=0}^{t} D_{ij}(t-\tau)\, \varepsilon_j^0(\tau). \tag{6}$$

In order to determine the vitual distortion $\varepsilon_i^0(\tau)$, for every successive instant τ, the following system of equations is solved:

$$\sum_j A_{ij}^0\, \varepsilon_j^0(t) = \begin{cases} (1-\mu_i)\, \overset{\mathrm{L}}{\varepsilon}_i(0) \ \text{for}\ t=0, \\ (1-\mu_i)\left[\overset{\mathrm{L}}{\varepsilon}_i(t) + \sum_j \sum_{\tau=0}^{t-1} D_{ij}(t-\tau)\varepsilon_i^0(\tau)\right] \\ \qquad\qquad\qquad \text{for}\ t>0, \end{cases} \tag{7}$$

where the matrix: $A_{ij}^0 = \delta_{ij} - (1-\mu_i)D_{ij}(0)$ is not time-dependent.

3. Damage Identification Methodology

Damage identification leads to the problem of minimization of the objective function F with respect to the vector of structural modification parameter μ_i . We assume the objective function as the distance between the original response (unmodified structure) and the observed one (damaged):

$$F(\mu_i) = \sum_\alpha \sum_t \left[f_\alpha(t,\mu_i) - \overset{\mathrm{M}}{f}_\alpha(t)\right]^2. \tag{8}$$

The gradient of the above function, with respect to μ_i, yields:

$$\frac{\partial F(\mu_i)}{\partial \mu_j} = 2\sum_\alpha \sum_t \left[f_\alpha(t,\mu_i) - \overset{\mathrm{M}}{f}_\alpha(t)\right] \frac{\partial f_\alpha(t,\mu_i)}{\partial \mu_j}, \tag{9}$$

where:

$$\frac{\partial f_\alpha}{\partial \mu_j}(t,\mu_i) = \sum_i \sum_{\tau=0}^{t} \breve{D}_{\alpha i}(t-\tau)\, \frac{\partial \hat{\varepsilon}_i}{\partial \mu_j}(t,\mu_i).$$

In order to determine the minimum of the objective function it is necessary to compute the distortion gradient $\frac{\partial \varepsilon_i^0}{\partial \mu_j}$. In consecutive iterations the vector of structural modifications μ_i can be determined as follows:

$$\mu_i^{(s+1)} = \mu_i^{(s)} - \frac{\frac{\partial F}{\partial \mu_j^{(s)}}}{\max|\text{grad}\, F|}\Delta, \tag{10}$$

where s denotes the current iteration, $s+1$ the next one and the step length Δ is adjusted due to the steepest descent optimisation strategy.

4. Single Defect Case

Let us consider a structure to be continually monitored. One (or more) permanently located sensor is collecting a response of the structure for a locally generated impulse. For the original (healthy) structure this response is considered as the reference one. If an analyzed response is different from the reference one it means that the structure has been modified. In this work we assume that only one defect occurred. In general case (multiple defects) for inverse analysis the whole strain influence matrix must be computed which is highly time demanding. The advantage of the proposed approach is minimization/decreasing of time computational. In the single defect case the reduced diagonal Impulse Influence Matrix is needed.

4.1. Investigation of the Objective Function in Single Element

In this section we will consider a single element of a structure as a possible defect location. For each single element an objective function is defined. A collection of the values of the objective functions forms a vector. Modelling the modification parameter μ_i^e by virtual distortion $\varepsilon_i^{0e}(t)$, the vector of objective functions is iteratively minimized. The least value/component of this vector indicates the damaged element. Thus the system of Eqns (7) can be written for each element e as follows:

$$A_{ij}^{0e}\, \varepsilon_j^{0e}(t) = \begin{cases} (1-\mu_i^e)\, \overset{\mathrm{L}e}{\varepsilon}_i(0) \ \text{for}\ \ t=0, \\ (1-\mu_i^e)\ \ \overset{\mathrm{L}e}{\varepsilon}_i(t) + \sum\limits_{\tau=0}^{t-1} D_{ij}^e(t-\tau)\, \varepsilon_j^{0e}(\tau)\ \ \text{for}\ t>0, \end{cases} \tag{11}$$

where: the matrices D_{ij}^e, $\varepsilon_j^{0e}(t)$ and $\overset{\mathrm{L}e}{\varepsilon}_i(t)$ concern an element e. For a beam element there are three components of strain, thus $i = 1,2,3$ and $j = 1,2,3$. Assuming one sensor is collecting response, the vector of objective functions F^e is built (for every element e):

$$F^e(\mu_i^e) = \sum_t \left[f_1(t,\mu_i^e) - \overset{\mathrm{M}}{f}_1(t)\right]^2. \tag{12}$$

For minimisation of the above objective functions we can rewrite Eqn (10) as follows:

$$\mu_i^{(s+1)} = \mu_i^{(s)} - \frac{\frac{\partial F}{\partial \mu_i^{(s)}}}{\max|\text{grad}\, F|}\Delta. \tag{13}$$

4.2. Numerical test

Let us consider a cantilever beam with the excitation applied to element No. 25, shown in Fig. 1. Load is time-dependent as marked in Fig. 2. The response of this beam is measured as difference of nodal rotations (element No. 44): $\overset{\mathrm{M}}{f}_1(t) = \alpha_2^{44}(t) - \alpha_1^{44}(t)$. The geometrical, physical and analysis data are given below:

- total length: $98cm$ (49 beam elements),
- cross section: $0.5cm \times 2cm$,
- density: $7800\,\frac{kg}{m^3}$,
- Young's modulus: $210\,GPa$,
- time step $3 \times 10^{-5}\ s$ (1000 steps) .

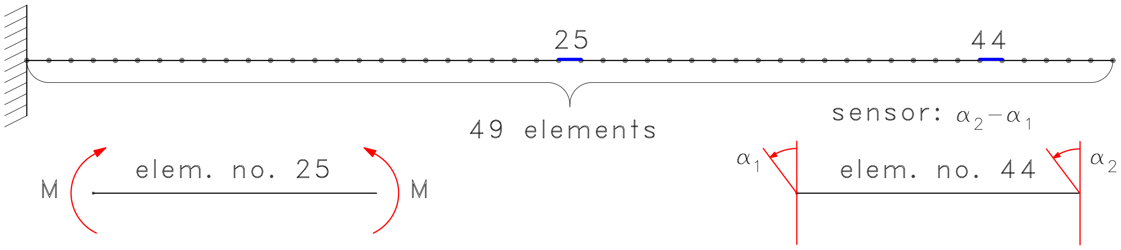

Figure 1: Tested cantilever beam.

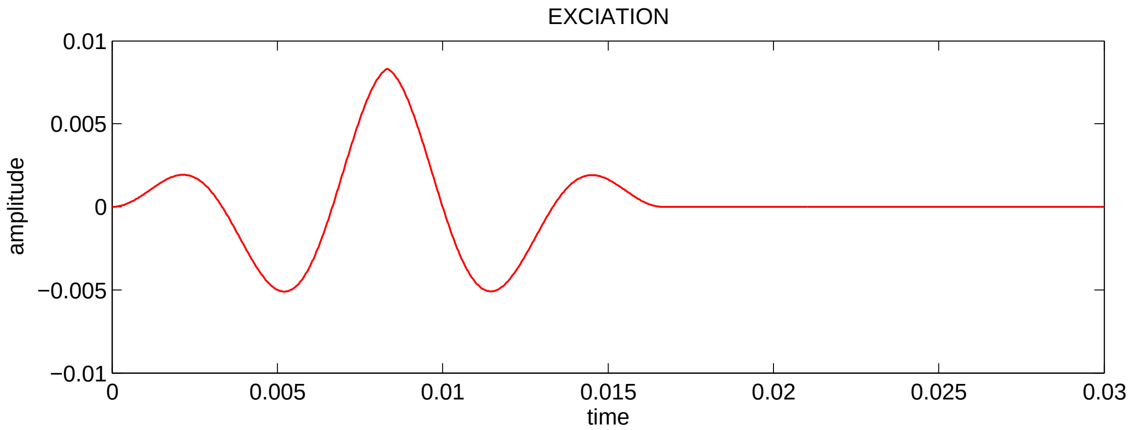

Figure 2: Beam excitation applied to element No. 25.

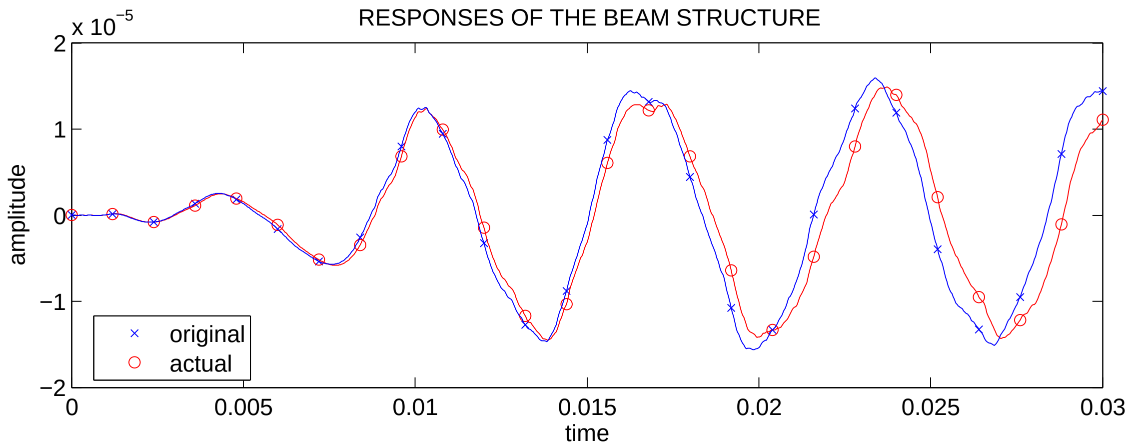

Figure 3: Beam responses measured by the sensor (element No. 44).

The measured response (in this case — calculated numerically) of the original beam is shown in Fig. 3. This graph also included the response of the modified structure — analysis was performed with the reduced Young's modulus of the element No. 40, $\mu_{40} = E/E' = 0.45$. In the inverse analysis it was assumed that possible defect can occur in elements Nos. $25 - 49$.

Figure 4 presents a decrease of the objective functions $f_i^{(20)}$ and $f_i^{(100)}$ obtained after 20 and 100 iterations respectively. The values of those objective functions are referenced to the first one $\frac{f_i^{(n)}}{f_i^{(1)}}$ (in the logarithmic scale). The most evident descent of the objective function ($\sim\ 10^{19}$ after 20 iterations and $\sim\ 10^{23}$ after 100 iterations) is for the element No. 40 and is indicating $\mu_{40} = 0.45$ (see Fig. 5). Identification history iterations and convergence of the objective function for this element are shown in Fig. 5 and Fig. 6.

4.3. *General Approach for a Single Defect*

In section 2 and 3 we discussed a general approach to inverse problem. Now, we will use the diagonal Impulse Influence Matrix for every time step t .

Let us consider a dynamically loaded structure with sensors placed in some locations similarly as in previous subsection. Let us assume, a possible defect to be detected is placed in element i and the measured response $\overset{\mathrm{M}}{f}_\alpha(t)$ is in location α. This response can be expressed by Eqn (5). In order to determine the generalized strain in location i with imposed virtual distortions we can use Eqn (6) substituting $j = i$ as follows (sum over i eliminated):

$$\varepsilon_i(t) = \overset{\mathrm{L}}{\varepsilon}_i(t) + \sum_{\tau=0}^{t} D_{ii}(t-\tau)\,\varepsilon_i^0(\tau). \tag{14}$$

On the other hand, the postulate, saying that the structure modelled by distortions and the modified one have to be identical in the sense of equality of their strain and stress fields, leads to the following equation:

$$\varepsilon_i^0(t) = (1-\mu_i)\,\varepsilon_i(t), \tag{15}$$

where the components of strain $\varepsilon_i(t)$ depend on distortions $\varepsilon_i^0(t)$. Substituting Eqn (14) to Eqn (15) we have the following relationship:

$$\mu_i \left[\overset{\mathrm{L}}{\varepsilon}_i(t) + \sum_{\tau=0}^{t} D_{ii}(t-\tau)\,\varepsilon_i^0(\tau)\right] = \\ = \overset{\mathrm{L}}{\varepsilon}_i(t) + \sum_{\tau=0}^{t} D_{ii}(t-\tau)\,\varepsilon_i^0(\tau) - \varepsilon_i^0(t). \tag{16}$$

From the above equation the virtual distortion $\varepsilon_i^0(t)$ can be found (cf. Eqn (7) and Eqn (11)):

$$a_i^0\varepsilon_i^0(t) = \begin{cases} (1-\mu_i)\,\overset{\mathrm{L}}{\varepsilon}_i(0) & \text{for } t=0, \\ (1-\mu_i)\ \ \overset{\mathrm{L}}{\varepsilon}_i(t) + \sum\limits_{\tau=0}^{t-1} D_{ii}(t-\tau)\varepsilon_i^0(\tau) & \text{for } t>0, \end{cases}$$

where the vector $a_i = \delta_{ii} - D_{ii}(0)(1-\mu_i)$.

The objective function and the vector of structural modifications can be used according to Eqn (8) and Eqn (10) ($j = i$).

5. Non-uniquess of the inverse problem

The non-uniquess of our inverse problem has been already studied and discussed in [4]. The VDM approach allows to identify the non-uniquess in advance and to modify the damage identification treatment accordingly.

Let us consider the tested truss-beam model (linear *waveduct*) shown in Fig. 7 with determined locations o for the tested impulse, i and j for potential damages and k for the sensor, respectively.

It has been discussed [3], [5], [6] that modelling of local stiffness reduction $\mu = E'/E$ (E denotes the initial stiffness and E' the modified one) introduced in locations i and j can be done through the following formulas:

$$\mu_i = \frac{\varepsilon_i(t) - \varepsilon_i^0(t)}{\varepsilon_i(t)} = \frac{\varepsilon_j(t) - \varepsilon_j^0(t)}{\varepsilon_j(t)}, \tag{17}$$

where $\varepsilon_i(t)$ and $\varepsilon_j(t)$ are superpositions of responses caused by an externally generated impulse (in location o) and responses due to locally generated virtual distortions ($\varepsilon_i^0(t)$ and $\varepsilon_j^0(t)$). It follows from Eqn (17) that the same modification μ located in element i or j leads to the relation (for each time step t):

$$\varepsilon_j(t)\,[\varepsilon_i(t) - \varepsilon_i^0(t)] = [\varepsilon_j(t) - \varepsilon_j^0(t)]\,\varepsilon_i(t), \tag{18}$$

or after simplification:

$$\varepsilon_j(t)\,\varepsilon_i^0(t) = \varepsilon_j^0(t)\,\varepsilon_i(t). \tag{19}$$

Also the following relations take place:

$$\varepsilon_i^0(t) = (1-\mu)\,\varepsilon_i(t), \qquad \varepsilon_j^0(t) = (1-\mu)\,\varepsilon_j(t). \tag{20}$$

Let us now postulate that the sufficient condition for non-uniquess of the inverse problem (for a symmetric configuration of elements o, i, j, k) is the identity of the following functionals:

$$\sum_{\tilde{t}\le t} D_{ki}(t-\tilde{t})\,D_{io}(\tilde{t}-\tau) = \\ = \sum_{\tilde{t}\le t} D_{kj}(t-\tilde{t})\,D_{jo}(\tilde{t}-\tau), \tag{21}$$

for every $\tau \le \tilde{t}$.

In order to demonstrate the related mathematical apparatus let us determine an auxiliary input distortion $\varepsilon_o^0(\tau)$ generated in location o and causing the same strain response in location i as locally introduced virtual distortion $\varepsilon_i^0(\tau)$:

$$\sum_{\tau\le\tilde{t}} D_{io}(\tilde{t}-\tau)\,\varepsilon_o^0(\tau) = \sum_{\tau\le\tilde{t}} D_{ii}(\tilde{t}-\tau)\,\varepsilon_i^0(\tau). \tag{22}$$

The proposed approach allows us to determine from the above equation the virtual distortions $\varepsilon_i^0(t)$ in each time step, creating local strain response (transmitted to the sensor located in element k) equal to the one generated by the input distortions $\varepsilon_o^0(t)$:

$$\varepsilon_i^0(\tilde{t}) = \frac{\sum\limits_{\tau\le\tilde{t}} D_{io}(\tilde{t}-\tau)\,\varepsilon_o^0(\tau) - \sum\limits_{\tau<\tilde{t}} D_{ii}(\tilde{t}-\tau)\,\varepsilon_i^0(\tau)}{D_{ii}(0)}. \tag{23}$$

On the other hand, the following two forms describing identical strains in location k can be proposed:

$$\varepsilon_k(t) = \sum_{\tilde{t}\le t} D_{ki}(t-\tilde{t})\,\varepsilon_i^0(\tilde{t}) = \sum_{\tilde{t}\le t} D_{kj}(t-\tilde{t})\,\varepsilon_j^0(\tilde{t}), \tag{24}$$

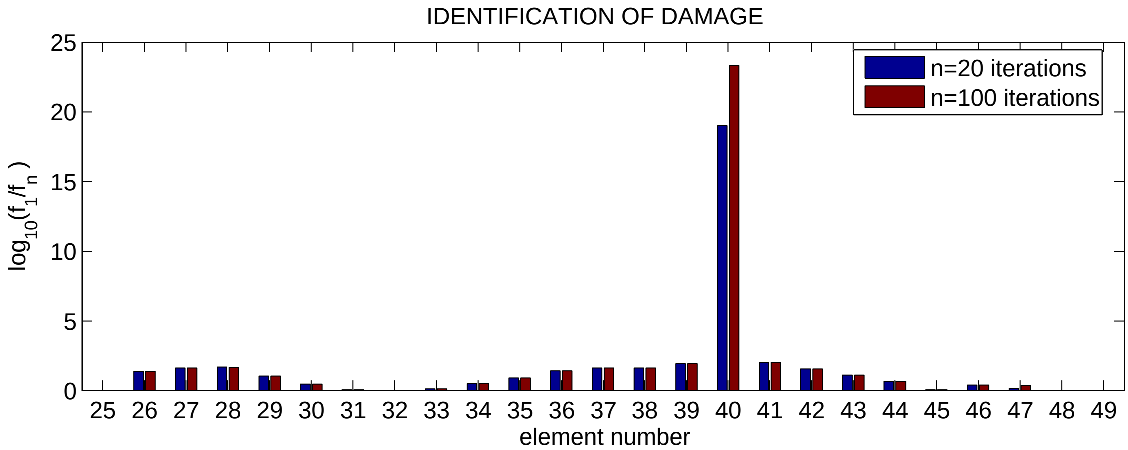

Figure 4: Damage identification — decrease of the objective functions (after 20 and 100 iterations) in the logarithmic scale.

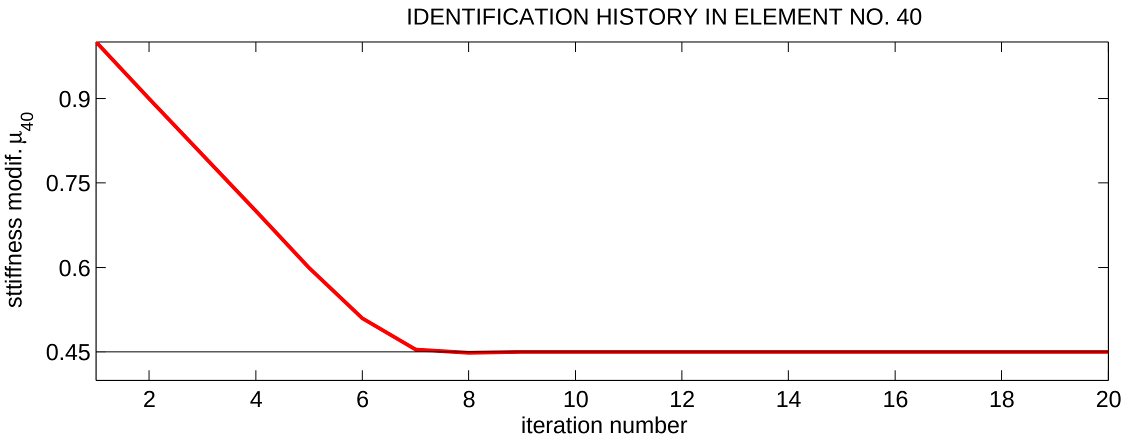

Figure 5: Identification history in element No. 40.

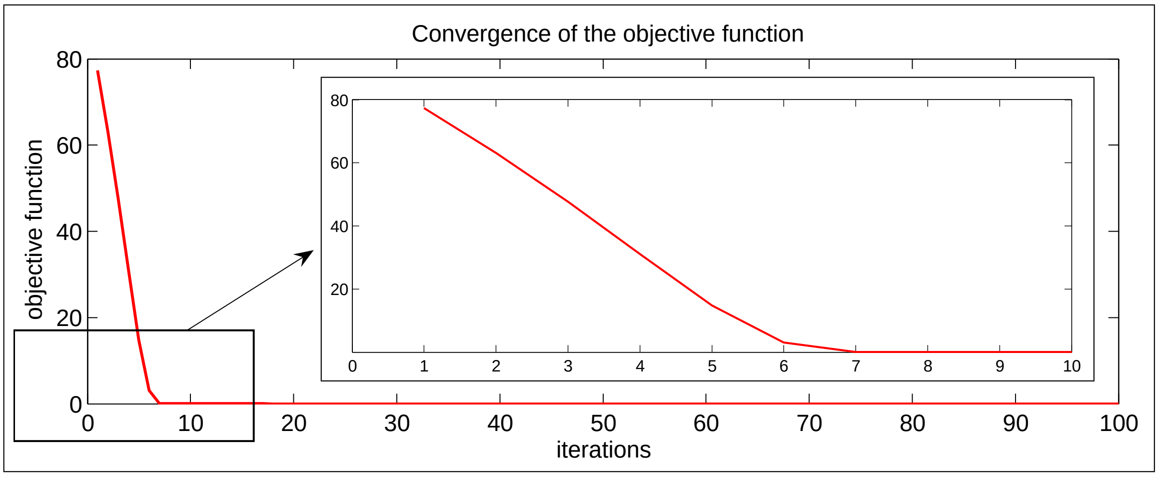

Figure 6: Convergence of the objective function in element No. 40.

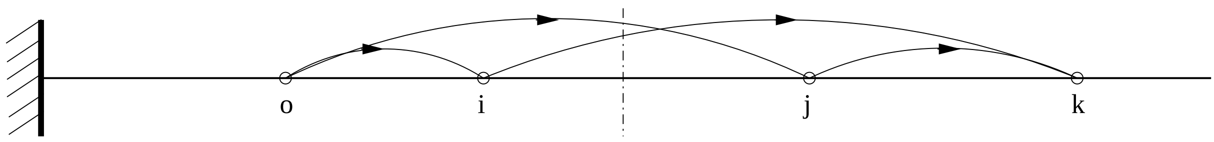

Figure 7: Scheme of description of wave propagation.

where $\varepsilon_i^0(\tilde{t})$ is determined by Eqn (23) and $\varepsilon_j^0(\tilde{t})$ by the analogous formula. Therefore, for given input distortions $\varepsilon_o^0(t)$, the structural response measured in location k can be computed in two ways from the formulas (23), (24).

The above result can be formulated symbolically in the following way:

$$D_{ki}\, D_{ii}^{-1} D_{io}\, \varepsilon_o^0 = D_{kj} D_{jj}^{-1} D_{jo}\, \varepsilon_o^0\,, \tag{25}$$

where D_{ii}^{-1} and D_{jj}^{-1} denote inverse operators for the influence functionals $D_{ii}(s)$, $D_{jj}(s)$. It can be shown that the following identity follows from the condition (21):

$$D_{ii} = D_{jj}\,, \tag{26}$$

for the functionals of our symmetrical case.

If local stiffness modifications take place in location i or j then the following modifications of structural response in location k is observed:

$$\Delta\varepsilon_k^i(t) = \sum_{\tilde{t}\le t} D_{ki}(t-\tilde{t})\, \varepsilon_i^{0\mu}(\tilde{t}), \tag{27}$$

or

$$\Delta\varepsilon_k^j(t) = \sum_{\tilde{t}\le t} D_{kj}(t-\tilde{t})\, \varepsilon_j^{0\mu}(\tilde{t}), \tag{28}$$

where $\varepsilon_i^{0\mu}(t)$ and $\varepsilon_j^{0\mu}(t)$ denote virtual distortions modelling parameter modifications (cf. (20)).

Lets us now demonstrate that for identical modifications $\mu_i = \mu_j = \mu$ introduced in locations i and j respectively (therefore, for valid condition (20) and for condition (21) from symmetry of the configuration), the output modifications (27) and (28) are also identical. To this end, let us describe these modifications of local strain responses due to $\varepsilon_i^{0\mu}(\tilde{t})$ and $\varepsilon_j^{0\mu}(\tilde{t})$ in the following two ways (making use of (20) and the left-hand-side expression of (22) applied to i and j):

$$\Delta\varepsilon_k^i(t) = \sum_{\tilde{t}\le t} D_{ki}(t-\tilde{t})(1-\mu) \sum_{\tau\le\tilde{t}} D_{io}(\tilde{t}-\tau)\, \varepsilon_o^0(\tau), \tag{29}$$

or

$$\Delta\varepsilon_k^j(t) = \sum_{\tilde{t}\le t} D_{kj}(t-\tilde{t})(1-\mu) \sum_{\tau\le\tilde{t}} D_{jo}(\tilde{t}-\tau)\, \varepsilon_o^0(\tau). \tag{30}$$

The above expressions describe modifications of responses (to the same excitation $\varepsilon_o^0(t)$) measured in location k, due to local stiffness modification μ placed in location i and j, respectively.

One can see, that the condition (21) implies the identity of the above response modifications $\Delta\varepsilon_k^i(t) = \Delta\varepsilon_k^j(t)$.

In conclusion, we have demonstrated that if condition (21) takes place for two locations i and j, then the same local stiffness modifications in these two locations lead to the same modifications of the structural response (measured in location k). Consequently, it leads to the non-uniquess of the inverse analysis and damage identification.

5.1. *Numerical example*

To illustrate the above discussion let us consider the numerical example shown in Fig. 1. One can check that the condition (21) is satisfied for the symmetrically located pairs of elements $10/40$, $11/39$, $12/38$, $\ldots 24/26$. For example, this identity for the pair of elements No. 19 and 31 is shown in Fig. 9 and identical time-dependent components $D_{22}(t)$ of the influence matrices for elements No. 19 and 31 are shown in Fig. 10. Different boundary conditions in two beam ends cause differences in the corresponding time-dependent components of the influence matrices $D_{22}(t)$ for pairs of elements $9/41$, $8/42$, $\ldots 1/49$. For example, functions $D_{22}(t)$ for the pair of elements $6/44$ are shown in Fig. 8.

Two identical signals measured in position k (element No. 37) as the response for the same excitation (Fig. 11) generated in location o (element 13) for the same damage of the intensity $\mu_i = 0.45$, placed in location i (element 19) or location j (element 31), are shown in Fig. 12.

The above result demonstrates that the inverse analysis has to lead to the non-unique solution of the damage identification problem (in the inner area of the beam between element No. 10 and element No. 40).

6. Summary

In general case, when several simultaneous defects are detected, the inverse analysis is not efficient enough. Therefore, in order to eliminate this problem, only one defect was assumed in the considered structure. In this case, the impulse influence matrix $D_{ij}(t)$ is reduced — the diagonal matrix (for each time step t) is computed, which is crucial for saving time and economical use of the memory. The methodology focused on detection and identification of only one defect is justified for continuous SHM installations. In this case one can assume that only one dominant defect can be generated in each time step.

In this paper it was shown that the presented inverse analysis (in particular cases) leads to non-unique solutions. It takes place if damages of equal intensity (also actuator and sensor) are localised symmetrically with respect to two ends of the presented beam. As a result of damage identification, an additional (false) defect is detected.

Practical conclusion is that in case of identical influence functions $D_{ii}(t)$ located on the diagonal of the influence matrix, the locations of sensor and actuator have to violate symmetry in order to avoid the non-uniquess of the inverse problem solution in SHM.

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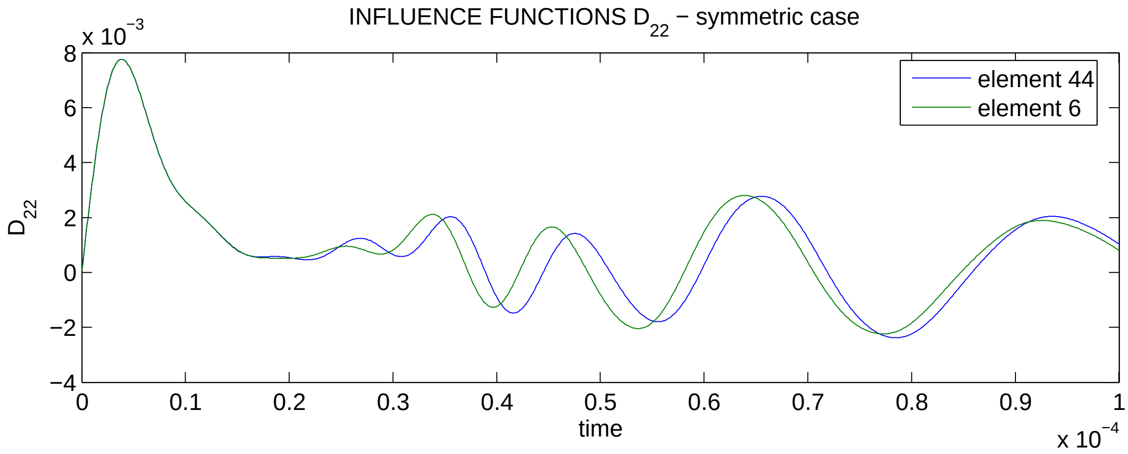

Figure 8: Comparisons of influence functions: $D_{22}(t)$ of the element No. 6 and 44.

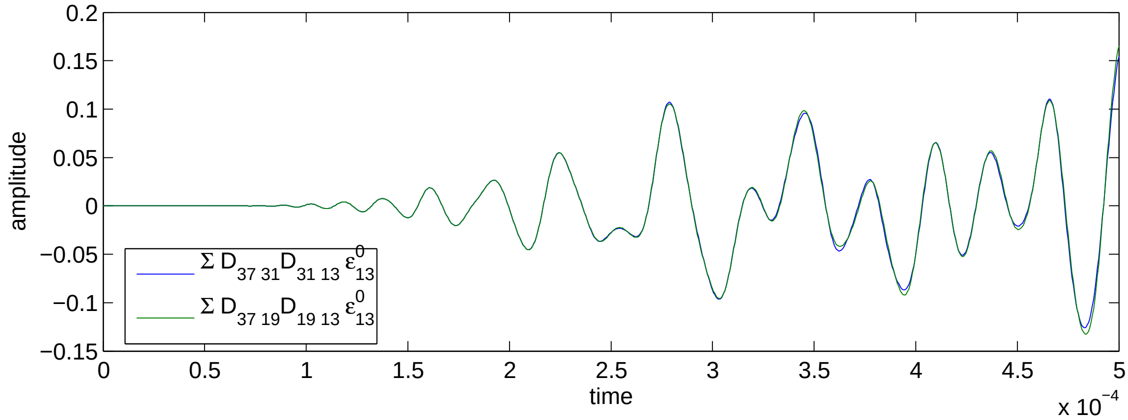

Figure 9: Comparison of products of the Eqn (21) for symmetric component of strain.

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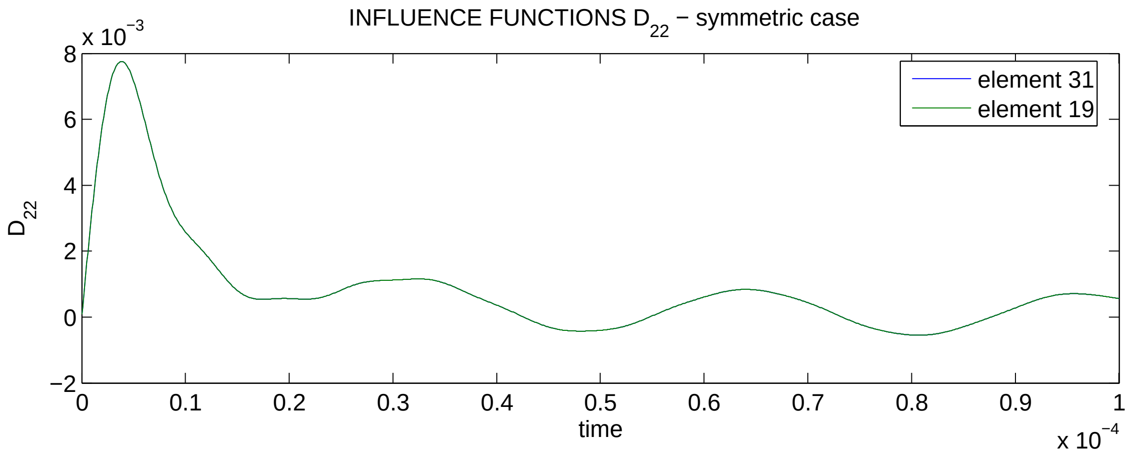

Figure 10: Comparisons of influence functions: $D_{22}(t)$ of the element No. 19 and 31.

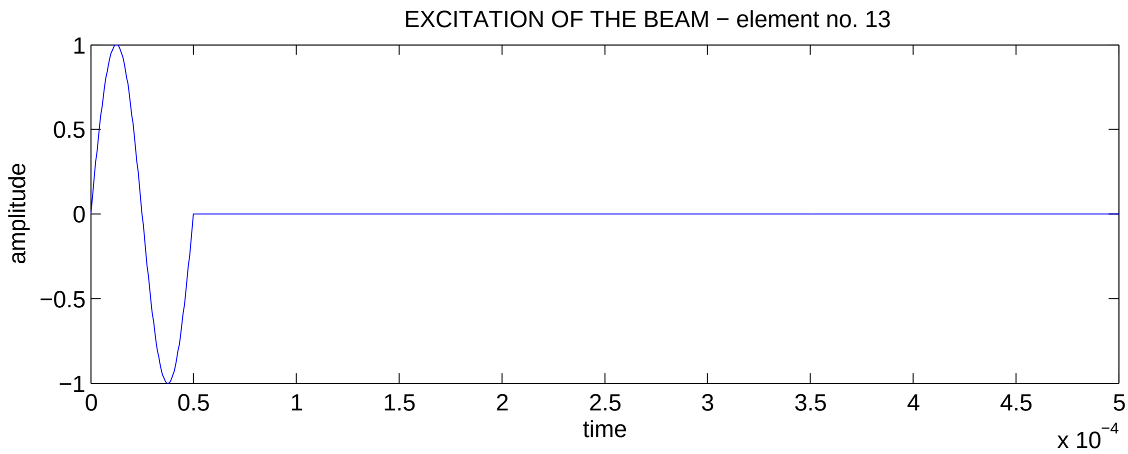

Figure 11: Excitation function (bending moment) applied to element 13.

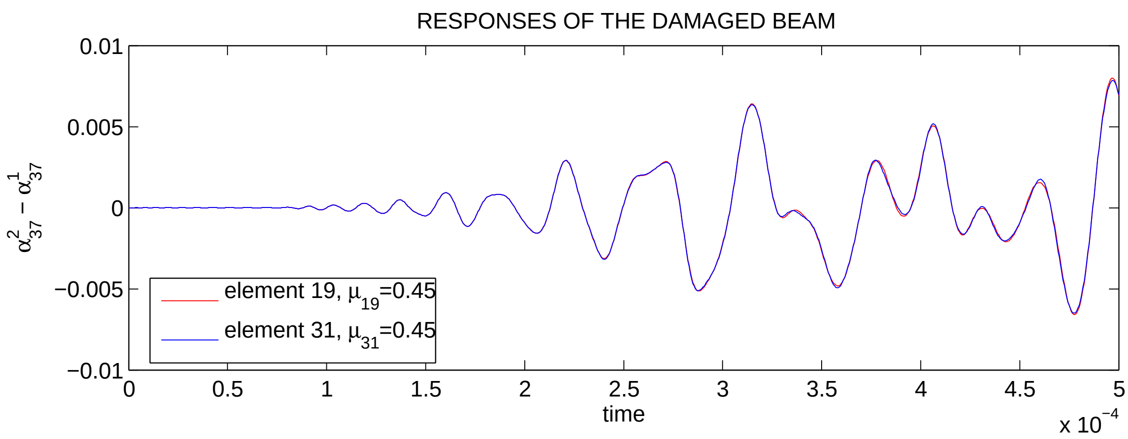

Figure 12: Measured responses of the damaged beam (element 19 and 31).